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FOR ALGEBRA I

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Journal Writing

Journal writing can be structured to give teachers cohesive and comparable information about students and their thinking while challenging them through contextual situations. A structure for journal writing includes prompts that focus on (1) mathematical content, (2) mathematical processes, and (3) student attitude or affect. Journal prompts give situations or questions to which students respond. Responses may include words, pictures or drawings, or symbols. Students are encouraged to support their ideas and to clearly explain what they mean. They can give specific examples as part of their explanations or use counterexamples.

Content prompts relate or connect topics within and outside of mathematics, targeting important or meaningful concepts and skills. They can also provide situations that focus on areas where students often have misunderstandings or misconceptions. The responses to the prompts give teachers (and students) insight into how a student has interpreted a mathematical idea.

Process prompts promote the awareness of how students solve or approach problems or algorithms. The responses to these prompts can give insight into students' preferences for problem-solving strategies or algorithms and into how they learn or remember. As students become aware of how they learn and solve problems, they grow more confident in approaching new or novel problems.

Attitudinal or affective prompts focus on students' feelings about themselves as mathematicians and students of mathematics. Students' responses allow teachers to assess how positive attitudes about mathematics and mathematicians are developing in the classroom environment.

Extended Problem-solving Tasks

Extended or expanded problem-solving tasks provide opportunities for students to explore and solve problems that require novel solution approaches. For this purpose, problem solving is defined as confronting a problem that does not have an obvious solution or solution path. In most cases, a non-routine solution method (or combination of methods) is required such as making a list, drawing a diagram, working backwards, guessing-and-testing, or creating a table.

Extended problem-solving tasks require more time and thought to solve than routine problems. Students draw on their previous knowledge and experiences to reason through the problem. Because their thought processes will be more complex, writing an expanded solution is an important part of communicating their methods or processes to others. Writing a response to an extended problem-solving task also helps students create a solution process as they clarify what the problem is asking, what information is given in the problem, and what solution methods would be appropriate.

Many students believe problem solving to be a linear process. That is, they read a problem, think of a solution method, solve the problem, and check their answer. Problem solving is more complex. It often requires re-reading a problem or abandoning one solution method for another.

Assessment Tasks Requiring Writing

Assessing student understanding can be done in a variety of ways including journal writing, homework problems, problem-solving write-ups, quizzes, and tests. Any assessment should encompass at least three types of tasks:

(1) problem solving, (2) conceptual understanding, and (3) skill acquisition.

Of the three types of assessment tasks, skill acquisition is most often assessed. These tasks would include solving equations and inequalities or using formulas by primarily symbol manipulation. Students often apply an algorithm that may or may not convey their understanding.

Items that are designed to assess students' conceptual understanding or ability to problem solve can provide a rich means by which students demonstrate their thinking and interpretations of concepts through expanded responses. The inclusion of these types of items link assessment with classroom practice. If students are required in mathematics classes to explain their thinking in class discussions or on their homework papers, it is important that assessments also include similar tasks. Likewise, if state assessments include self-constructed response items, students will develop skill in responding to such items when these types of tasks are regularly included on a daily basis as well as on assessments.

... and more

- ✎ There are 12 extended problem-solving tasks. Each task requires more time to solve than one class period. Students often provide the best solutions if they are given 10 days in which to solve it. The teacher may decide to use one of these every 3 weeks or so.
- ✎ In some cases, teachers may assign the tasks for the entire class to work individually. These tasks also give teachers and students the opportunity to use pair or group problem solving. Regardless, it is important that students write their responses in a way that a reader can see the flow of their thinking and understand the solution method or path that they used.
- ✎ It is recommended that students do extended problem solving on a regular basis. This practice supports their development of problem-solving strategies and boosts their confidence to solve complex problems.
- ✎ For each problem-solving task a solution has been given. However, there are multiple methods to solve each problem. Teachers should be open to creative ways that students may approach these problems.
- ✎ There are 10 assessment items included here. These items represent a conceptual approach to a particular mathematical topic. There should be no more than one of these items on a chapter test. If used independent of the chapter quizzes or tests, however, it is possible to use more than one. Additionally, any of the content or process journal prompts can be used on formative or summative assessments in a similar manner.
- ✎ It is important to recognize that students should have experience with talking and writing mathematics daily in order for the use of these assessment items to be representative of student understanding. If a teacher uses a lecture-based approach, some of these items may need alteration to allow for optimal responses from students.

Evaluating Students' Responses

Students tend to give the writing more thought if it is to be scored. One method used was to score the responses on content rather than on grammar and spelling. An essay grading method was used. By reading two or three papers to get a feel for students' responses these first papers formed the baseline for scoring other papers. This method, however, did not provide students a guideline for writing in advance.

When teachers use rubrics and metrics, however, scoring guidelines can be provided in advance. A rubric establishes qualitative levels that define what characteristics a response has or what criteria it meets. The qualitative levels are not used as points. On the other hand, a metric is a point system. The levels of a metric have specific criteria associated with each point value. The same criteria may be used for either a rubric or a metric system. The difference is in whether or not points or qualitative levels are established for the response.

There are many ways to create a rubric or metric. Teachers can develop one alone, or students can work with the teacher to develop a rubric or metric specific to their class or to the task. Sample rubrics and metrics, developed with students' participation, follow on the next page.

Although grammar and punctuation are not usually scored, students should communicate their ideas in ways that are understandable. Some students may use drawings, tables, charts, or other means to convey their ideas. They should be encouraged to use whatever ways they need to make their ideas clear.

As writing tasks are used, excerpts can be taken from students' papers to illustrate qualities that you consider important. Both high- and low-quality responses can be used to show students the comparison with rubric or metric criteria. Of course, authorship of whatever responses are selected should be kept anonymous.

Rubrics or metrics to score the problem-solving tasks as extended types or as assessment items can be developed for each individual task or created as a general guide for student performance. The rubrics or metrics that follow can also be adapted to serve as a generalized set of criteria to guide students' solution approaches. If rubrics or metrics are being used by the entire mathematics department, it may be appropriate to have department-wide discussions to agree upon criteria. This will provide a means by which to motivate consistent and cohesive student work across grades, courses, and teachers.

With any form of writing and any type of rubric or metric, students can self-evaluate their responses or conduct peer evaluations. This allows you to see if students truly understand the criteria outlined in the rubric or metric. This activity also requires students to think at a much higher level as they analyze critically others' work .

General Metric

4 points The student's work includes—

- ✎ completed prompt or an answer to the question posed
- ✎ support for statements made by using either examples or counterexamples
- ✎ ideas clearly communicated to the reader
- ✎ legible writing, drawings, pictures, charts or tables, and diagrams
- ✎ accurate mathematics or information

3 points Omission of one criterion from level 4

2 points Omission of two criteria from level 4

1 point Omission of three criteria from level 4

0 points Omission of more than three criteria from level 4

Three-level Rubric (or Metric)

The student's work shows a response that—

Exceeds standard

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader

Meets standard

- ✎ addresses the question raised in the prompt
- ✎ has some correct or accurate mathematics
- ✎ is legible
- ✎ does not support or justify some of the statements made
- ✎ makes sense to the reader

Below standard

- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not support or justify statements made

Five-level Rubric (or Metric)

4 The student's work shows a response that—

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader.

3 The student's work shows a response that—

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ does not have fully justified or supported statements
- ✎ makes sense to the reader

2 The student's work shows a response that—

- ✎ addresses the question raised in the prompt
- ✎ has some incorrect or inaccurate mathematics
- ✎ is legible
- ✎ does not have justified or supported statements
- ✎ is somewhat clear to the reader

1 The student's work shows a response that—

- ✎ addresses the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is partially legible
- ✎ does not have justified or supported statements
- ✎ does not make sense to the reader

0 The student's work shows a response that—

- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not make sense to the reader

Implementing Journal Writing in Your Classroom

- ✍ To begin using writing in your classroom, you will need to make sure your students understand your expectations for writing. The following offers one method for helping students learn what is meant by *writing in mathematics*.
- ✍ Share with students the rubric or metric you will be using. You may opt to create a rubric or metric with your students rather than creating one yourself. Make copies of the rubric or metric for students to keep in their notebooks. Post one copy in the classroom for easy reference.
- ✍ Give students a practice prompt to write. If it is used as a warm-up, allow about 6 minutes for them to respond. The practice prompt can be any type, but the rubric or metric may work better with a content prompt. Select one that you feel all students in your class can attempt.
- ✍ Have students compare what they wrote with their partner or table mates. They should check the rubric or metric. Have students focus on 3 things they could do to improve their writing to the next higher level. If their writing already includes all the indicators for the top level, ask them to write another question that this prompt made them think about.
- ✍ Allow 3 minutes for students to correct or revise their work. They should strive to reach the top two levels of the rubric or metric.
- ✍ Collect the work. Score it with your rubric or metric. However, to allow students time to learn to meet your expectations, you may not want to record the score yet.
- ✍ For the next several days, whether you assign prompts as a warm-up or for homework, allow students time to revise or correct their work. You should stress that they should strive to reach the top two levels. Repeat this phase as often as needed to help students understand your expectations for their writing.
- ✍ If possible, show students samples of other students' writing. Use this sample to illustrate what you mean by your criteria in the rubric or metric. A sample of student writing in the middle grades follows.

Sample of Student Work

Prompt: Corey said, “Division is commutative.” Do you agree with Corey? Why or why not?

○	<p>No, I don't agree with Corey. He must be thinking about multiplication or addition because division is not commutative. If it was commutative, it would mean that you can put the numbers you're dividing in any order and you can't. If you put $4 \div 2$ you get 2. But if you put $2 \div 4$ you get one half and that's not the same.</p>
	<p>In multiplication you can put the numbers in any order and still get the same answer. Like if you multiplied 3×2 or 2×3, it doesn't matter. You still get 6.</p>
	<p>I think Corey messed up and really meant to say that division is NOT commutative. He should be more careful about his work.</p>

Content prompts

Number Sense, Properties and Operations with Real Numbers

- 1. Jenna said, “I get the same product when I multiply 12.8×48 as well as when I multiply 128×4.8 .” “That makes sense,” said Thad. Why would Thad say this? Verify that this is true.**

Students may include a discussion about the equivalency of the two expressions. If one factor is increased by a factor of 10 and the other factor by a factor of 0.10, the product will remain the same. Some students may multiply to show that the product is the same but encourage them to focus on the generalization of the mathematics.

- 2. “Which problem has the larger quotient?” asked Chelsea. “ $\frac{1}{2} \div 4$ or $2 \div \frac{1}{4}$?” Decide which one you believe has the larger quotient. Justify your answer without computing the quotient.**

Students should include reference to the meaning of division with fractions. Some may say that division can mean that you measure the dividend by the divisor. Thus in the second problem, $\frac{1}{4}$ ‘fits’ eight times in 2 while 4 only ‘fits’ into one-half $\frac{1}{8}$ times. Thus the second problem gives the larger quotient.

- 3. “I think subtraction is not commutative,” said Josh. “I agree,” said Corey, “but I can think of a time when order does not matter in subtraction.” What do you think Corey meant? Can you find an instance when order does not matter? Explain.**

Students should describe an instance when the minuend and the subtrahend are the same numbers. This is the only time when order does not matter.

- 4. Ciara wrote, “The quotient of two integers is always an integer.” Kristen read the statement. Then she said, “Is that always true?” What do you think? Justify your idea(s) with details.**

It is not always true, only if the dividend is a multiple of the divisor.

- 5. “I wonder,” said Katelin, “if the difference of two whole numbers is always a whole number.” What would you tell Katelin? Justify or support your answer.**

This is only true if the whole-number subtrahend is less than the whole-number minuend. If the subtrahend is larger, then the difference will be an integer.

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- 6. “I found out something last night,” said Sammi. “A number raised to an odd power is always odd.” Do you agree with Sammi? Why or why not? Support your answer.**

If two odd numbers are multiplied, the resulting product is odd. By using the third power, you would first multiply the number times itself. In the case of an odd number, this would give an odd number. The next multiplication, for the third power, will still yield an odd product. If the base or factor is even, multiplying it times itself would give an even number. Thus the next multiplication would also give an even number. Therefore Sammi’s statement is only true if the base or number is odd. Students should disagree with Sammi.

- 7. “I can take any number and raise it to a third power. The result will always be a positive number,” said Anisha. Is that true? Why or why not? Support your answer.**

This is not true. If the base or number is 0 or an integer, the third power will not be a positive number.

- 8. Jeremy wrote the following on the board.**

$$4^2 \square 4 \square 4 \square 4 = 76$$

Use any operational symbols in the boxes to make the number sentence true. Justify your answer by describing your process.

A possible solution is $(4^2 + 4) \times 4 - 4 = 76$. Students should relate the solution to the order of operations.

- 9. Find two ways to write 64 as a number raised to a power. Justify your answer.**

Students may consider 2 or 4 as a base such as 2^6 or 4^3 . Some may use 2 raised to a power as a base such as $(2^2)^3$. Another response may be 8^2 .

- 10. Erica said, “ -2^4 is -16 .” Eric argued, “No, -2^4 is 16.” Who do you agree with? Why?**

Students should agree with Erica and support their answer. Some students may use order of operations to support their answers.

- 11. Rex said the opposite of a number is the same as its additive inverse. Randy said they are not the same. With whom do you agree and why?**

Agree with Rex. The sum of a number and its additive inverse is 0, which is also true for opposites.

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- 12. When Steve simplified $(-3)^4$, he got 81. Sharon got -12 . Do you agree with either solution? Why or why not?**

Agree with Steve. Sharon computed -3×4 rather than using -3 as a factor four times.

- 13. Place parentheses in the expression to make a true statement. Justify your answer.**

$$5 \cdot 2 + 3 \cdot 4 = 22$$

$(5 \cdot 2) + (3 \cdot 4) = 22$. Some justification should be given.

- 14. Glen wanted to compute the sales tax on a shirt that cost \$21 and a pair of shorts that cost \$17. If the tax is 7%, show Glen two ways he could compute the tax correctly. Describe both ways to him.**

Students typically use the distributive property to find the two ways. The tax can be computed on the amounts separately and then added OR the tax can be computed on the total cost.

- 15. Find five pairs of numbers whose product is 1. Write the multiplication sentences. What patterns do you notice?**

Students should notice that the numbers are reciprocals or multiplicative inverses. These patterns can introduce a discussion about these topics.

- 16. April said, " $4^{-2} \times 4^0$ equals 4^0 because you multiply exponents." Leigh said, "I think it equals 4^{-2} ." Do you agree with either one? Why or why not?**

Students should agree with Leigh. They may give a variety of reasons, but most frequently they note that 4^0 is 1, or they use the law of exponents related to multiplying with like bases.

- 17. Solve $4 \cdot 23 \cdot 25$. Describe your process.**

Solve it another way. Describe that process.

Compare the two results. What conclusions can you make?

Students may first solve it in the order written. Another way is to group 4 and 25 together first, to make an easier problem. Or students may multiply 23 and 25 first. Their conclusions will vary depending on which methods they elect to use.

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- 18. Rhonda said $\frac{8}{15} \div \frac{2}{3}$ was solved by dividing 8 by 2 and 15 by 3. Her quotient is $\frac{4}{5}$. Do you agree or disagree with her method? Why?**

This prompt will provide some interesting looks at fractional division. New algorithms may be developed.

- 19. “Why is it that when you multiply two fractions between 0 and 1 the product you get is smaller than the two fractions you multiplied?” asked Derek. “Well,” said Cedric, “I can tell you.” What do you think Cedric said? Support your answer.**

Students may use a variety of ways to justify this generalization. They may use a rectangular model to explain it with area. Or they may provide multiple examples and generalize their findings. Other methods are possible.

- 20. Keila asked, “How can I estimate the product of $82.34110792483 \times 5.1$?” “I know an easy way,” said Justin. What do you think Justin told Keila? Describe Justin’s process without computing. Be specific.**

Students may use the front-end method of estimation. Their estimate may vary by the numbers that are chosen. For example, one student may use 82×5 and then indicate that the actual product is larger than this because 82 and 5 are both smaller than the actual numbers. Other students may use 80 and 5 or 80 and 5.1 with similar discussions.

- 21. Ms. Atkins told her class that addition and subtraction and multiplication and division are opposite operations. What did she mean by that? Be specific in your explanation.**

Addition and subtraction (multiplication and division) can be thought of as opposite operations because one can ‘undo’ the other. Students may present informal explanations that show this or others may present a formal proof.

- 22. Find four pairs of numbers whose sum is 0. Write the addition sentences for your pairs. What do you notice?**

The pairs are additive inverses. If zero is one of the numbers, then the pair is made up of two zeros.

- 23. Marty dropped spaghetti sauce on his math paper. What do you think is under the sauce he dropped? Why?**

$$\frac{2}{0} = \text{[sauce blotch]}$$

Division by 0 is undefined and students should give a rationale as to why it is undefined.

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- 24. Megan challenged Amanda to find the larger quantity without using a calculator or computing it with paper and pencil.**

Which is larger: 6^9 or 9^6 ? How do you know?

6^9 is the larger amount. Students may have various ways of determining which one is larger.

- 25. Courtney said she could use the distributive property of multiplication over addition to help her multiply mixed numbers. Here is what she wrote:**

$$\begin{aligned}3\frac{2}{3} \times 4\frac{1}{2} &= (3 \times 4) + \left(\frac{2}{3} \times \frac{1}{2}\right) \\ &= 12 + \frac{1}{3} \\ &= 12\frac{1}{3}\end{aligned}$$

Is her method correct? Why or why not?

Her method is not correct. She did not apply the distributive property appropriately. Thus her product is too small.

- 26. Myrna says all numbers have reciprocals. Nicky disagrees. Who is right? Why do you think so?**

Not all numbers have reciprocals. Nicky is correct. Zero does not have a reciprocal.

- 27. Jensen said, “The square root of a number is always smaller than the radicand.” What do you think? Why?**

This is not true. As one counterexample, the square root of one-fourth is one-half. One-half is greater than one-fourth.

- 28. Kevala noticed that $\frac{7}{\sqrt{7}}$ is equal to $\sqrt{7}$. How can you explain this?**

The original fraction was multiplied by $\frac{\sqrt{7}}{\sqrt{7}}$ or 1 to obtain the radical.

- 29. Chester wrote $\sqrt{25} = -5$. Lester wrote that $\sqrt{25}$ is impossible. Who is right? Why?**

Chester is incorrect because -5 raised to the second power would be 25. Lester is correct if imaginary numbers cannot be used.